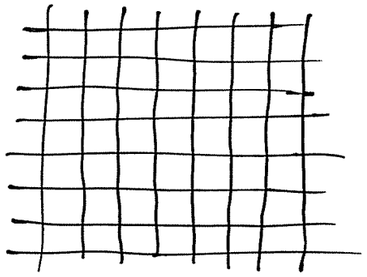
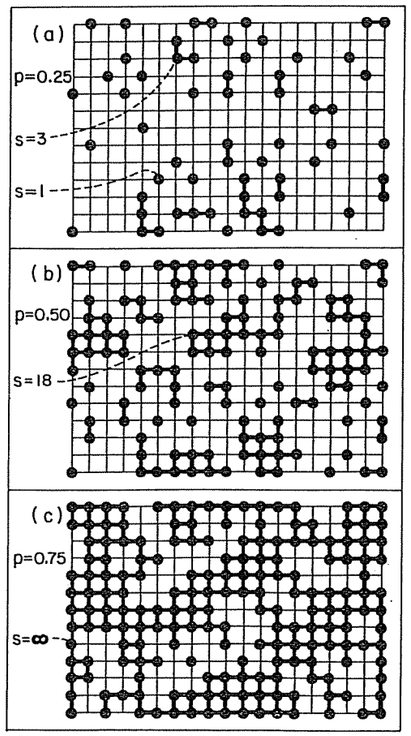
Appendix: Percolation Problem

- A geometrical problem with critical phenomena
- Consider a square lattice

Site percolation problem 

p: Probability that a lattice site is occupied by "•"



Site percolation on the square lattice, illustrating various cluster sizes (s) for three values of p, the fraction of filled sites. For $p = 0.75$, an unbounded cluster is present.

← small p: small clusters of neighboring dots of different sizes s

← Some $p \approx p_c$: at the edge of emergence of an "infinite" cluster

← large p: An "infinite" or "percolating" cluster that spans the lattice exists (plus some clusters)

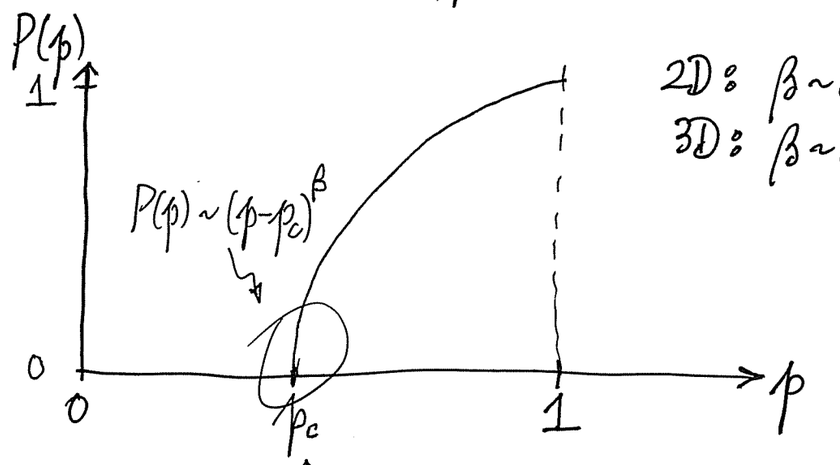
Question

- $P(p)$ = prob. that a randomly chosen site belonging to the percolating cluster

  cluster that goes from one side of the lattice to another side

- $p=0$: No percolating cluster $\Rightarrow P(p=0)=0$
- $p \ll 1$: Only small finite clusters $\Rightarrow$ No percolating cluster $\Rightarrow P(p \ll 1)=0$
- $p=1$: Full lattice is percolating cluster $\Rightarrow P(p=1)=1$

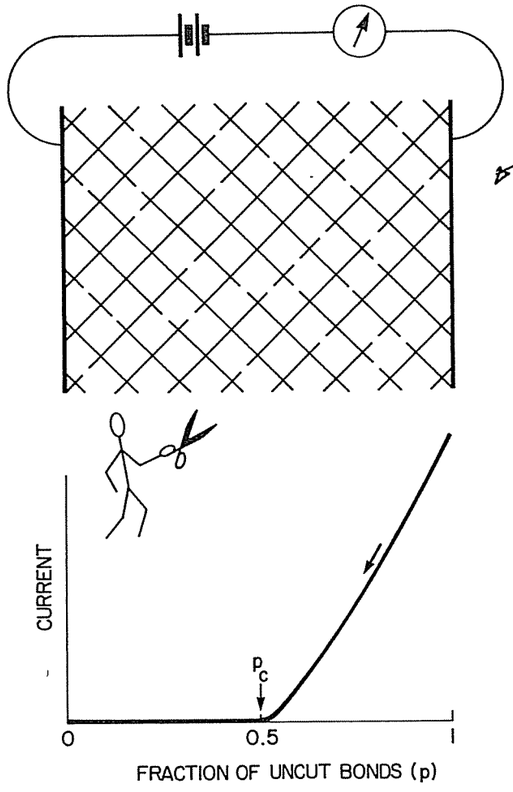
→ Some transition happens in between



↑ Value of p_c depends on dimension of lattice

Many other interesting questions

- Distribution of cluster sizes as a function of p
- Is there a "Hamiltonian" representing percolation problem?
- Putting a transport problem (conductance) on to percolation problem



The randomly cut network as an example of percolation.

randomly cutting conducting links

"bond percolation"

[meaning: link]

Below p_c: No conducting path from left to right

$G_T \equiv$ Conductance of whole lattice

$G_T \sim (p - p_c)^t$

[Figures taken from R. Zallen, "The Physics of Amorphous Solids"]

- Mean field theories
- More accurate theories & Numerical simulations
- Fractal properties of percolating cluster near p_c
- Variations:
  - Continuum percolation (no underlying lattice)
  - percolation in complex networks
  - Applications

Refs:

- R. Zallen, "The physics of amorphous solids" for a very readable (a bit old) introduction
- K. Christensen and N.R. Moloney, "Complexity and Criticality" for a chapter on percolation introducing the topic at the MSc level rather thoroughly.